THERMAL SCREENING EFFECTS IN A FERROMAGNET

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In considering the theory of spin waves,[1] we have examined elementary excitations of the Hamiltonian

$$H = -\sum J(R_{ij})\vec{S}_i \cdot \vec{S}_j \tag{1}$$

and lowest-order interactions among these excitations, in cases when $J(R_{ij})$ is a very long-ranged, oscillatory interaction.[2] As numerical computation is required, we are preparing to program the IBM 7094 for this project. In the course of this preparation, we have, however, obtained a result which is independent of the details of the calculation. This result appears to have several interesting consequences, and we have therefore made it the subject of the present communication.

What we have found, to put it simply, is that there exists a sort of "Debye length," $R_0(T)$ (to be defined below), which separates the interactions into a "near zone" and a "far zone," and that although the thermal effects on the interactions in the near zone must be found numerically, the only thermal effect in the <u>far</u> zone is a uniform screening. That is, the interactions $J(R_{ij})$ for values of $R_{ij} \gg R_0(T)$ appear to be multiplied by a universal factor

$$M(T)/M(0), \tag{2}$$

where $M(T)$ is the magnetization at temperature T, and $M(0)$ its value at absolute zero. We have also been able to extend the Dyson[3] form of the power series expansion for $M(T)$ to the actual case of nonnearest-neighbor interactions, as well as the absence of a term in T^3 and the presence of a term in T^4. From our equations it is seen that not only the magnitude but also the sign of each coefficient in the Dyson expansion is a function of the range of the interaction. This is in accord with qualitative arguments of Keffer and Loudon,[4] but in disagreement with results of the Green function factorization technique of Bonch-Bruevich and Tyablikov,[5] which is presumably in error.

The shielding effect which we have found in our examination of the interaction must also be active in screening magnetic impurities of all sorts, scattering neutrons, etc., and is a manifestation of transverse spin correlations. The excitation of spin waves shields the transverse components of the spin vectors so that far away all that is seen is the average z component of the spins. At short distances, what has been called "energy renormalization"[4] is found to be the correct procedure, whereas only at distances long compared to $R_0(T)$ is the "magnetization renormalization"[5] correct.

The present results were obtained strictly in the context of nonlinear spin-wave theory recently introduced by Bloch[6] in the study of nearest-neighbor interactions in the simple cubic lattice. We have rederived[1] her starting Hamiltonian and find that it appears to be at least qualitatively accurate in the lower range of temperature $T < T_c$. We therefore postulate the validity of this method in our proof, which is furthermore restricted to (a) spins 1/2, (b) cubic structures, and (c) crystals vastly larger than the range of the interaction. However, the qualitative results are so simple and so suggestive that it is tempting to conjecture they are more valid than the restricted proof we offer, and that (2) is always the correct long-range diamagnetic screening constant in a ferromagnet of type (1).

The spin-wave energy at $T=0$ is[2]

$$\epsilon_k = \sum_i J(R_i)[1 - \cos\vec{k}\cdot\vec{R}_i] \simeq k^2/2\mu + O(k^4), \qquad (3)$$

where the sum extends over all lattice points, and μ, which appears in the leading term in a Taylor series expansion of ϵ_k, is denoted the "mass" of the spin wave. Using the results of reference 6, we find for the effective spin-wave energy[7] $\epsilon_k(T)$ the set of coupled equations,

$$\epsilon_k(T) = \sum J_T(R_i)[1 - \cos\vec{k}\cdot\vec{R}_i] = k^2/2\mu(T) + O(k^4),$$

where

$$J_T(R_i) = J(R_i)[1 - \delta_T(0) + \delta_T(R_i)] \qquad (4)$$

and

$$\delta_T(R_i) = \frac{2}{N} \sum_{k \in \text{B.Z.}} \frac{\cos\vec{k}\cdot\vec{R}_i}{\exp[\beta\epsilon_k(T)] - 1}, \quad \beta = \frac{1}{k_B T}. \qquad (5)$$

From this one sees that $\frac{1}{2}\delta_T(0)$ equals the number of spin waves thermally excited, and that the magnetization therefore decreases with temperature according to the formula,

$$M(T) = M(0)[1 - \delta_T(0)]. \qquad (6)$$

We shall now prove that for $R_i \gg R_0(T)$ the variable term $\delta_T(R_i)$ in Eq. (4) tends to zero, so that only the constant factor $1 - \delta_T(0)$ remains, and the simple proportionality

$$J_T(R_i) \simeq J(R_i)M(T)/M(0) \quad [R_i \gg R_0(T)] \qquad (7)$$

relates the effective interaction $J_T(R_i)$ to its value $J(R_i)$ at absolute zero.

We define the thermal length

$$R_0(T) \equiv [\pi^2\beta/8\mu(T)]^{1/2} \qquad (8)$$

in analogy with the thermal diffusion length of nondegenerate free electrons. It follows that since the only wavelengths which can significantly contribute to $\delta_T(R_i)$ are those for which

$$\beta\epsilon_k(T) \ll 1, \qquad (9)$$

the sum can be evaluated to lowest order in this small parameter. In the far zone, the asymptotic behavior is simply [for $R_i \gg R_0(T)$]

$$\delta_T(R_i) \simeq \frac{2}{(2\pi)^3} \int d_3k \, \frac{\cos\vec{k}\cdot\vec{R}_i}{k^2} \frac{2\mu(T)}{\beta}$$

$$= \tfrac{1}{8}\pi \frac{1}{R_i R_0^2(T)} \to 0, \qquad (10)$$

which is all that is required to prove Eq. (7) and establish the desired result. We now make some remarks based on this.

We consider it significant that near absolute zero, the thermal length approaches infinity as $T^{-1/2}$, and therefore <u>all</u> interactions are <u>well within</u> the near zone.[8] This makes it valid, at sufficiently low temperature, to expand the cosines, and obtain the low-temperature behavior of Eq. (5):

$$\lim_{T\to 0} \delta_T(R_i) \simeq \frac{2}{N} \sum_{k\in \mathrm{B.Z.}} \frac{1-\frac{1}{2}(\vec{k}\cdot\vec{R}_i)^2}{\exp[\beta\epsilon_k(T)]-1} + O\left(\left[\frac{R_i}{R_0(T)}\right]^4\right). \tag{11}$$

The self-consistent solution of Eqs. (4) and (11) leads to the terms $T^{3/2}$, $T^{5/2}$, $T^{7/2}$, and finally T^4, in the power-series expansion of $M(T)-M(0)$ near absolute zero. The coefficients must be obtained numerically from the function $J(R_i)$, but aside from this the present results are sufficiently similar to those of references 3, 4, and 6 that we need not elaborate further.

On the other hand, the Green function method[5] gives for the spin-wave energy in the absence of an external field,

$$\epsilon_k(T) = [M(T)/M(0)]\epsilon_k, \tag{12}$$

a too simple relation which although similar at first sight to what we have found [see Eq. (7)], is incorrect in detail, and is, in fact, tantamount to assuming that the thermal length vanishes (instead of which it actually becomes infinite at $T=0$). Moreover, this relation leads to a T^3 term in the low-temperature expansion of the magnetization[5] referred to previously, in disaccord with current notions about this state of affairs; all of which does not prevent the Green function method from perhaps giving a satisfactory account[9] of the thermal properties near to and above T_c, where spin-wave theory is sure to fail; but this remains to be shown.

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[1] With a view to applying nonlinear spin-wave theory to the study of the thermal properties of rare earth metals, and the calculation of T_c.

[2] For example, the Ruderman-Kittel interaction: M. A. Ruderman and C. Kittel, Phys. Rev. 96, 99 (1954). See also D. Mattis and W. Donath, Phys. Rev. 128, 1618 (1962), and further references therein. The possibility that spin-wave theory may not be applicable has been discussed [D. Mattis, Phys. Rev. 130, 76 (1963)], and such cases are, of course, excluded from the present considerations.

[3] F. J. Dyson, Phys. Rev. 102, 1217, 1230 (1956). Only nearest-neighbor interactions are considered therein.

[4] F. Keffer and R. Loudon, Suppl. J. Appl. Phys. 32, 2 (1961).

[5] V. L. Bonch-Bruevich and S. V. Tyablikov, The Green Function Method in Statistical Mechanics (North-Holland Publishing Company, Amsterdam, 1962), Chap. VII, and references therein.

[6] Micheline Bloch, Phys. Rev. Letters 9, 286 (1962).

[7] $\epsilon_k(T)$, $\mu(T)$, and $J_T(R_i)$ are the generalizations to finite temperature of the corresponding ground-state quantities ϵ_k, μ, and $J(R_i)$.

[8] Therefore the constant shielding effect we have found is probably not valid at small distances (except near the Curie temperature), whereas at large separation it may be observable at low (but always nonvanishing) temperature.

[9] The Green function method is being extended by many investigators beyond the results[5] of Bonch-Bruevich and Tyablikov. In fact, K. Kawasaki and H. Mori, Progr. Theoret. Phys. (Kyoto) 28, 690 (1962), have shown how a higher order decoupling leads to Eqs. (4) and (5) of the text.